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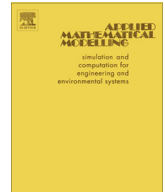
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Power supply–demand balance in a Smart Grid: An information sharing model for a market mechanism



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ABSTRACT

In the future, global energy balance of a Smart Grid system can be achieved by its agents deciding on their own power demand and production (locally) and the exchange of these decisions. In this paper, we develop a network model that describes how the information of power imbalance of individual agents can be exchanged in the system. Compared to existing network models with hierarchical structures, our developed model, together with a market mechanism, achieve the power balance in the system in a completely distributed way. Additionally, dynamics, constraints and forecasts of each agent can be conveniently involved.

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1. Introduction

The Smart (Power) Grid is the most promising solution for the problems presented by increased electrification, and the large-scale introduction of distributed power generation in the power system.

The Smart Grid offers a number of significant advantages. First, the Smart Grid allows for two-way communication, which enables demand response. Secondly, domestic power generation is a key component, which makes the end-user both a producer and a consumer, or a prosumer, of electric power [1]; in a Smart Grid, prosumers are both incentivized and empowered to contribute to the balance of power supply and demand in the system. Thirdly, by producing power locally, Smart Grids also minimize transportation costs. A problem with the transportation of electricity is found in the fact that energy is lost in the power network transmission lines. Matching supply and demand at a local level therefore can be used to better minimize the losses from transportation; a feature of the Smart Grid which offers both economic and environmental gains.

Another important feature of the Smart Grid is found in the fact that if local matching lowers the fluctuations in the power system, Smart Grids will ease the control effort of the overall power system [2]. However, because the end-user decides when to use his electric devices, a major question that arises here is: how do we coordinate the decisions of a large number of end-users? In the power system, the end-users can have a large variety of electric power demand and production devices, such as washing machines, freezers and micro combined heat and power systems, which can be controlled even if they are subject to operational constraints. The rest of the power demand, that cannot be controlled, can, to some extent, be predicted. This means that the decisions have to be coordinated on two levels. Firstly, the end-user has to anticipate on the forecasted power profile. Secondly, since power is shared in the network, the end-user must also anticipate on how neighbors decisions will influence the power profile. Therefore, in order for the end-users to contribute to the system in

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an optimal way, a difficult control problem has to be solved to coordinate the decisions. As the network grows in the number of end-users, a large number of decision variables has to be included in the optimal control problem.

It is widely agreed that a centralized solution scheme for the so-called optimal control problem is too time consuming, because of the computational complexity [3]. Therefore, a host of scalable control methods have been proposed in the Smart Grid setting. Current models, proposed by the literature for device coordination, have a centralized component in the fact that there is one decision-making agent, see e.g. [4]. In the PowerMatcher game [5] for example, an agent for each device broadcasts a bidding curve for his willingness to pay for electricity. One agent at the top of a hierarchical structure, then determines the equilibrium price. The PowerMatching concept was implemented in Groningen, in the Netherlands, as a demonstration project of a future energy-infrastructure called PowerMatching city, see [6]. Twenty-five households with smart appliances, such as micro-combined-heat-power systems that match their energy use in real time based upon the available energy generation, were connected. The project was generally perceived as a big success, however a number of short-comings were observed. In particular, predictions of power is not yet taken into account, and since the prices are the same everywhere in the network, there is no preferred location for the production in the network.

This observation motivates us to consider a fuller model, with a distribute information structure for scalability, as well as including predictions on the power demand. In [4] a methodology combining forecasts, planning and real time control, that is capable of distinguishing position in the network, is described. However, the planning is centralized. Probably the most related to the results presented in this paper is the work presented in [7], where a multi-agent Model Predictive Control (MPC) approach is presented. However, the method is applied to load frequency control, which is a different type of problem than what we treat and, most importantly, an information sharing structure has not been considered. In particular, we address the challenge to match local supply and demand real time anticipating on the future behavior and only base decisions on local information. Further, we will avoid a hierarchical network structure.

In this paper, and to avoid a centralized structure, we propose an information network where each agent has local (imbalance) information about the system when they make their decisions. Further, in order to anticipate on future behavior, and to incorporate constraints from the electric devices, we work in a MPC framework. In the MPC framework we include predictions about the end-users future power demand, and technical constraints from the devices that need to be controlled, see e.g. [8–10]. We propose an information-sharing network, and dynamically couple the end-users information to coordinate decisions in the network. The information at an end-user is a mixture of personal imbalance, and the connected neighbors imbalances. In a large network, the distance between suppliers and consumers also plays a role: it is more energy efficient to buy from a close-by end-user, than a far-away end-user. An end-user cannot exchange imbalance information with everybody, but bargains directly with a subset of all end-users in the network. This motivates the choice of information network topology. The idea is that the system, as a total, reaches the same balance as if it could bargain with all end-users directly, but now there is an ordering by information distance to neighbors of who an end-user buys his power from: if the power is available at the direct neighbors, the end-user will buy from this neighbor, and the power coordination is done locally. In the case that an end-user needs to buy from a neighbor that is not a direct neighbor, he must bargain through his neighbors neighbor connections until an end-user wants to sell.

The information network which the proposed model models, is made up by a subset of the end-users in the power network, which is connected to the overall power system. This means that there is also a power exchange between the sub-network and the external network. However, this exchange is not modeled explicitly, but the objective is formulated as if the members of the information network are forming a closed grid, minimizing the imbalance, meaning that the power exchange with the external network is minimized. Then, after the actions are taken, we assume that the excess or shortage of power is taken care of by the external network. The control goal is the supply demand balance at a market level within the information network. We stress that the end-users are virtually connected to the information network, while they are physically connected to the power network. Therefore, the information network does not need to have the same topology as the power network, but it can have any desired topology.

In this way, it is clear that coordinating decisions in the information network will influence the control of central power plants. In the literature, the Optimal Power Flow (OPF) problem is solved to find the optimal power generation given line power constraints. This is a steady state optimal control problem. In contrast to our formulation, the objective of the OPF is to minimize generation cost while the balance problem is included as a hard constraint. In [11], a dynamic feedback controller for an optimal real-time update was designed. However, predictions and anticipation on the future situation in the network can not easily be included.

Our proposed information sharing model facilitates distributed decisions of dynamically coupled prosumers in a Smart Grid with input constraints. We take into account forecast about future behavior when the decision is made. If the end-user receives local real time information concerning the systems status, possibly in the form of prices, he can make decisions for when to turn on or off his demand, production or storage of electric power such that both the end-users and the overall system benefits. Such a control strategy using a price mechanism is described in [12,13] where it has been applied to a vehicle formation example. The strategy is based on dual decomposition and sub-gradient iterations, and [14] describes a distributed model predictive control (d-MPC) version of the method. This way, the end-user can make his control decision based on price incentives from virtually connected neighbors, local imbalance information predictions, his own constraints and his own predictions. In [15], which focuses on the control aspects of this approach for Smart Grids, the positive effect of the scalability of the distributed algorithm was shown, and in general distributed grids are more robust to topological failures [16].

The main contributions of this paper are found in the concept of an information model in the power network, and how to explicitly design the information sharing network taking into consideration the low, medium and high voltage network. The model can be used with a completely distributed MPC algorithm to coordinate decisions in the complex power network, and the model handles constraint from different types of electric devices and can have any information topology.

The rest of this paper is organized as follows. Section 2 develops the dynamic models of the end-users in the network, and includes discussion of the necessary properties of the information exchange in the network. Section 3 gives examples on how to build an information matrix for a power network. Section 4 presents the main idea for solving the optimal control problem completely distributed, and Section 5 shows a simple example on how our model works together with the d-MPC method to coordinate households with μ -CHP. Finally, Section 6 discusses the results of this paper.

2. Modeling

In this section we develop our model for the coordination of power production and consumption in a multi-producer multi-consumer Smart Grid. We expand the description given in [15], and include flexible production and demand. The goal for the agents is to minimize the power imbalance in the network, which corresponds to adjusting their demand and production to balance the network.

2.1. System description

We start by describing the dynamics of power imbalance $\tilde{x}_i(k)$ of an agent $i = 1, \dots, n$ in a network of n agents at time k . An agent (prosumer) is for example a household with a micro Combined Heat and Power system and other electric devices such as washing machines, freezers and batteries of electric cars, where the electrical production and demand can be adjusted or shifted in time. Each agent has a power demand $d_i(k)$ and a power production $p_i(k)$, and we distinguish between flexible power demand $d_{f,i}(k)$ and production $p_{f,i}(k)$ and external power demand $d_{e,i}(k)$ and production $p_{e,i}(k)$. With the following relations

$$\begin{aligned} d_i(k) &= d_{f,i}(k) + d_{e,i}(k), \\ p_i(k) &= p_{f,i}(k) + p_{e,i}(k). \end{aligned} \quad (1)$$

In this case, the external demand and production are all the power demand and production at the agent that can not be adjusted. The external signal can be measured at each time step k , while the flexible demand and production can be adjusted by the agent.

The agent decides when to turn on or off, and how much to ramp up or down the flexible power devices, i.e. it chooses the change in flexible power demand $u_{d,i}(k)$ and production $u_{p,i}(k)$, given by

$$\begin{aligned} u_{d,i}(k) &= d_{f,i}(k+1) - d_{f,i}(k), \\ u_{p,i}(k) &= p_{f,i}(k+1) - p_{f,i}(k). \end{aligned} \quad (2)$$

At the same time, it measures the change in external demand $w_{d,i}(k) = d_{e,i}(k) - d_{e,i}(k-1)$ and production $w_{p,i}(k) = p_{e,i}(k) - p_{e,i}(k-1)$.

Remark 1. For simplicity we include one demand-side device and one supply-side device in the equations, but it is straight forward to include more devices.

The physical power imbalance $\tilde{x}_i(k)$ caused by agent i is given by the imbalance at the previous time-step plus the change in flexible power $u_i(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} u_{d,i}(k) \\ u_{p,i}(k) \end{bmatrix}$ and change in external power $w_i(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} w_{d,i}(k) \\ w_{p,i}(k) \end{bmatrix}$

$$\tilde{x}_i(k+1) = \tilde{x}_i(k) + u_i(k) + w_i(k), \quad (3)$$

where $i = 1, \dots, n$ and $\tilde{x}_i(k), u_i(k), w_i(k) \in \mathbb{R}$. In (3), each agent i keeps track of its own imbalance. However, the agent needs to share some information with neighbors in the network in order to sell or buy power among the other agents.

We include information exchange in the network through dynamic coupling between the agents' notion of imbalance information. Each agent i has a state Eq. (5) also involving imbalance information from neighboring agents $j \in N_i$, where

$$N_i \subseteq \{1, \dots, n\} \setminus \{i\}, \quad (4)$$

is agent i 's set of information neighbors from which agent i receives information at a time step k , and agent i itself is excluded in this set. The model for the imbalance information $x_i(k) \in \mathbb{R}$ at agent i is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + B_{ii}u_i(k) + w_i(k), \quad (5)$$

where B_{ii} is the input weight, A_{ii} weighs the power imbalance information of agent i itself, and A_{ij} weighs the information received from its neighbors $j \in N_i$. We choose the initial values of $x_i(0)$ to be the real physical imbalance of agent i at the initial time, i.e. $x_i(0) = d_i(0) - p_i(0)$ and $w_i(0) = 0$.

Remark 2. When $x_i(k)$ is a scalar, we will see in Section 4 that this will result in one price (Lagrangian multiplier) associated with each agent. If one wishes to associate one price with the power demand and one price with the power production, the power demand and production information $x_{d,i}(k), x_{p,i}(k)$ needs to be kept separated and $x_i(k) = [x_{d,i}(k) x_{p,i}(k)]$. Another choice is to let $x_i(k)$ have the length of the number of flexible devices at the agent, in which case each device gets a price associated.

Notice that in (5) the *physical* imbalance enters the system at each agent i through change in flexible power $u_i(k)$ and change in external power $w_i(k)$, where the physical imbalance is included in the *information* about imbalance $x_i(k+1)$. As time evolves, information spreads through the network through the neighboring agents N_i . In this way close-by information neighbors can react faster to a change in external power $w_i(k)$ than information neighbors further away. In Fig. 1 agent 2 is a close by information neighbor of agent 1 while agent 5 is a far away information neighbor of agent 1.

If we define the vectors

$$\begin{aligned} x(k) &= [x_1(k), \dots, x_n(k)]' \in \mathbb{R}^n, \\ u(k) &= [u_1(k), \dots, u_n(k)]' \in \mathbb{R}^n, \\ w(k) &= [w_1(k), \dots, w_n(k)]' \in \mathbb{R}^n, \end{aligned} \quad (6)$$

where prime means transpose, then the compact form of the model given in (5) is

$$x(k+1) = Ax(k) + Bu(k) + w(k), \quad (7)$$

where input matrix $B \in \mathbb{R}_+^{n \times n}$ is diagonal since all agents have a flexible power input, they can only control their own input, and *information matrix* $A \in \mathbb{R}_+^{n \times n}$ specifies the topology and weighs the information flow in the network.

The non-zero elements of information matrix A can be defined with the notion of a graph. We define a directed graph $D = (H_n, E_n)$, with n agents. The agent set is given by $H_n = \{1, \dots, n\}$, and $E_n \subseteq H_n \times H_n$ denotes the edge set. There is an edge in the graph whenever information is communicated directly from agent i to agent j , i.e. $A_{ij} \neq 0$ ($j \in N_i$) if and only if $(i, j) \in E_n$. Fig. 1 shows an example for $n = 5$.

Further we have three more restrictions for how to choose the elements of the information matrix. First of all we require the elements A_{ij} to be such that the total imbalance in the network of the uncontrolled system with no change in demand is time invariant. Thus

$$\sum_{i=1}^n x_i(k+1) = \sum_{i=1}^n x_i(k), \quad \forall k \geq 0, \quad (8)$$

when $w(k) = u(k) = 0$, $\forall k \geq 0$. Secondly, we only consider non-negative elements in the A matrices, since we view the imbalance as a quantity that we want to divide between the agents. Third and finally, for stability of the uncontrolled system, we require the spectral radius of A to be less than or equal to one.

Condition (8) implies the following straightforwardly obtained requirement:

Proposition 3. The column sums of information matrix A are equal to one, i.e.,

$$\sum_{i=1}^n A_{ij} = 1, \quad j = 1, \dots, n. \quad (9)$$

Proof. This is readily checked by substituting (5) into the left hand side of (8) for $w_i(k) = u_i(k) = 0$ for all i and k , since $A_{ij} = 0$ when $j \notin N_i$. \square

Corollary 4. If A is a irreducible non-negative matrix and (9) is valid, we are ensured that the spectral radius is one.

Proof. Since A is a stochastic matrix, the result follows directly from the Perron–Frobenius Theorem, e.g. [17]. \square

Corollary 4 implies that we must choose our information graph to be strongly connected so that the system (7) is stable when $w(k) = u(k) = 0$ for all k . Notice that there is still design flexibility in the A matrix, even when the above requirements are met.

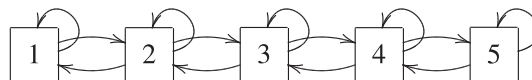


Fig. 1. A graph with five agents and a tri-diagonal A matrix. The arrow from agent i to agent j indicates the direction of information flow. Self-loops come from diagonal elements of A .

In the end we relate the physical imbalance at the agents to the imbalance information at the agents. With the above requirements on the information weights A_{ij} and initial conditions $x_i(0) = \tilde{x}_i(0) = d_i(0) + p_i(0)$, summing over all agents for (3) and (5) results in the relation

$$\sum_{i=1}^n x_i(k) = \sum_{i=1}^n \tilde{x}_i(k), \quad \forall k \geq 0, \quad (10)$$

which means that the total imbalance information in the network is equal to the total power imbalance in the network even though $x_i(k) \neq \tilde{x}_i(k)$ at an agent level.

3. Constructing the information matrix A

We have argued that the information topology and the power network topology does not need to be equal. In this section, however, we will build an information matrix A motivated by the physical structure of the power grid to demonstrate that this is possible as well. Fig. 2 displays a schematic overview of the Dutch power infrastructure, where a circle represents an agent. Then we continue to motivate for a more scarce information topology, and even a possible change in the power grid structure itself, using our network model.

There is a high voltage (HV), a medium voltage (MV), and finally a low voltage (LV) transmission network to which the agents are connected. The reason for this layered structure is to minimize losses in the power lines. The losses in the lines increase with the current. By increasing the voltage the current for transporting the same amount of energy can be decreased.

However, households in The Netherlands need to be connected to a grid of 220 Volt. To transform down the voltages between the different layers of transmission networks there are transformer stations, indicated as squares in Fig. 2. These transformer nodes have load constraints, if the load over the transformer is too high a power black out occurs. Since network components have to be designed for the peak-load, it would be good for the network if the load is kept close to a target value. This way the network is used in a more efficient way. The circles in Fig. 2 represents the n number of agents described in Section 2.1. These LV connections are prosumer households.

In order to construct one row in the information matrix $A \in \mathbb{R}^{n \times n}$, we start by looking at one agent i , represented by a dark colored circle i in Fig. 2, and discuss how to weigh the neighbours $j \in N_i$. Suppose that a change in power demand occurs at agent i present in a street in city X, for instance by turning on a washing machine. Then, it is natural that if agent i has available production capacity, he should ramp up the production so that the additional power imbalance added to the network by agent i is as low as possible. It is also clear, that physical neighbors of agent i in the same street and connected to the same LV network, should ramp up their production before a distant physical neighbor in a far away city Y would do so. This is because we wish to keep the load over a transformer station low, or at a target value. With these considerations, we will first assume a full information matrix, i.e. $N_i = n - 1$ for all $i = 1, \dots, n$. In this example, we assume that there is one HV network, with $v \in \mathbb{N}_+$ number of HV-MV transformer stations. Each of the $i = 1, \dots, v$ MV networks has $\eta_i \in \mathbb{N}_+$ number of MV-LV transformer stations, where η_i varies from MV network to MV network. Each LV network has $m_i \in \mathbb{N}_+$ number of LV connections, i.e. m_i number of agents. The number of agents in the system is therefore $n = \sum_i m_i$.

Suppose agent i , represented by one of the dark colored circles in Fig. 2, weights his own power imbalance with a factor $\alpha \in \mathbb{R}_+$, and he weights all neighbors in the neighborhood connected to the same LV transformer, the rest of the dark in the figure, by $\beta \in \mathbb{R}_+$. Then choosing $\alpha > \beta$, reflects that agent i reacts heavier on his own imbalance than his LV neighbors. Or equivalently, agent i finds it more important to react to the change of his own state, than to react to changes in neighboring agents' states. Next, agent i weights all the agents at a different LV network, but the same MV network, represented by striped circles in Fig. 2, by a weight $\epsilon \in \mathbb{R}_+$. In the end distant agents, maybe located in the other side of the country, at the same HV network but different MV network is given a lower weigh $\epsilon' \in \mathbb{R}_+$. These agents are denoted by white circles in Fig. 2. Agent i organizes the relative importance of different type of neighbors by $\alpha > \beta > \epsilon > \epsilon'$.

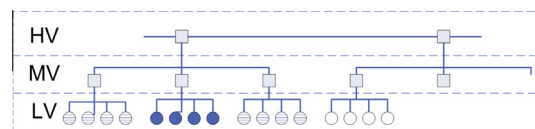


Fig. 2. Schematic representation of the current power network in the Netherlands. The squares represent transformer nodes, and the circles represent end agents.

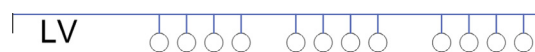


Fig. 3. Possible grid topology, where a large cluster of agents are locally balanced.



Fig. 4. Possible information structure.

If we assume that all agents in the power network make the same choices, we can build an information matrix $A \in \mathbb{R}^{n \times n}$ where all column sums equal one, by blocks of sub-stochastic matrices $A_{LV}^{(i)}$, $A_{MV}^{(i)}$, and one matrices $G_{LV}^{(i)}$, $G_{MV}^{(i)}$ of proper dimensions by

$$A_{LV}^{(i)} = \begin{bmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{bmatrix} \in \mathbb{R}^{m_i \times m_i}, \quad (11)$$

$$A_{MV}^{(i)} = \begin{bmatrix} A_{LV}^{(1)} & \epsilon G_{LV}^{(2)} & \cdots & \epsilon G_{LV}^{(\eta_i)} \\ \epsilon G_{LV}^{(1)} & A_{LV}^{(2)} & \cdots & \epsilon G_{LV}^{(\eta_i)} \\ \vdots & & \ddots & \vdots \\ \epsilon G_{LV}^{(1)} & \epsilon G_{LV}^{(2)} & \cdots & A_{LV}^{(\eta_i)} \end{bmatrix} \in \mathbb{R}^{\eta_i \times \eta_i}, \quad (12)$$

$$A = A_{HV} = \begin{bmatrix} A_{MV}^{(1)} & \epsilon' G_{MV}^{(2)} & \cdots & \epsilon' G_{MV}^{(v)} \\ \epsilon' G_{MV}^{(1)} & A_{MV}^{(2)} & \cdots & \epsilon' G_{MV}^{(v)} \\ \vdots & & \ddots & \vdots \\ \epsilon' G_{MV}^{(1)} & \epsilon' G_{MV}^{(2)} & \cdots & A_{MV}^{(v)} \end{bmatrix} \in \mathbb{R}^{v \times v}, \quad (13)$$

where $\mathbb{R}^{\eta_i \times \eta_i}$ and $\mathbb{R}^{v \times v}$ are the dimension of the number of blocks in the corresponding $A_{MV}^{(i)}$ matrices and A matrix. The dimension of A in terms of agents is $\mathbb{R}^{n \times n}$, and the parameters $\alpha, \beta, \epsilon, \epsilon'$ are chosen such that $\sum_j A_{ij} = 1$, meaning that no information is lost.

In this setup, the A matrix (13) is full, meaning that agent i receives information from all neighbors in the network. However, with our way of modeling (5), we are not restricted to choose the physical neighbors to be communication neighbors. We can freely choose our communication neighbors, which has the consequence that neighbors in a LV transportation grid may have different information about the situation in the grid, when they make decisions about the power they will produce (buy). If no agent implements any action, all agents in the network will converge to see the same imbalance if A is chosen according to Corollary 4.

3.1. Other possible grid configurations

By taking into account all elements in (13) the information exchange is not easily manageable for a large system. A strength of the framework is that we can set elements in the A matrix to zero, representing information that is not important to an agent. We then have a smaller number of direct neighbors, and thus less information exchange. The remaining elements of A should be updated accordingly, as the model is valid for any structure of the A matrix such that Proposition 3 holds, $A_{ij} \in \mathbb{R}_+$, and A is a matrix such that the system is stable.

Above, the current network structure was captured in the model, but with a large enough share of distributed generation the hierarchical structure in Fig. 2 may not be necessary. One possible configuration is given in Fig. 3. In the extreme case, a large number of agents can form a stand-alone power grid.

The corresponding connectivity matrix for Fig. 3, can be built up by matrix (11). The A_{LV} matrix has the dimension of the number of neighbors connected to one LV transformer.

In Fig. 3 the physical and the information structure is still the same. However, we can also adjust the number of information neighbors, so that the information infrastructure does not correspond to the physical infrastructure in the network. The physical structure may be as in Fig. 3 while the information structure is a chain as in Fig. 4. All agents then take exactly two neighbors into account in the update of their state equation. When the graph is strongly connected, and if no agent implements any action, all agents in the network will now still converge to see the same imbalance. An agent i in city X then only needs to obtain information about change in demand from a few neighbors in his own street. If there is a change in demand at an agent j in a distant city Y, this information will only reach agent i if the imbalance is not already taken care of.

Claim 5. By choosing physical close neighbors as information neighbors, local production for local demand is stimulated. This is because the agents that receive the information about a change in demand the fastest, will be able to react to this change first. Consequently transportation losses in the power grid are avoided.

4. The optimal control problem

The goal is to find the control inputs $u_i(k)$ for all agents $i = 1, \dots, n$ such that the imbalance $x_i(k)$ becomes zero for all agents $i = 1, \dots, n$, given the influence from neighbors and physical constraints from the devices. Due to the constraints,

we work in the Model Predictive Control (MPC) framework, see e.g. [8–10]. This means that instead of finding an explicit control law, a finite time optimization problem is solved iteratively at each time step k .

In addition, we want to distribute the computation effort, so that each agent makes his decision only based on local information. Therefore, we briefly explain the distributed Model Predictive Control (MPC) method to find the change in flexible power $u_i(k)$, for $i = 1, \dots, n$, in a completely distributed manner. A central MPC problem is decomposed into substantially smaller sub-problems. Each sub-problem is iteratively solved independent of each other and combined into a global solution. The details on how the distributed MPC method relates to the centralized MPC method can be found in [14,18,15]. The idea is shown in Fig. 5.

For each agent i , given an imbalance $x_i(k)$ and change in flexible production $u_i(k)$, we associate an objective function $V_i(x_i(k), u_i(k))$. In particular we choose the objective function at time k to be

$$V_i(x_i(k), u_i(k)) = R_{ii}x_i^2(k) + [u_{d,i}(k) \quad u_{p,i}(k)]Q_{ii}\begin{bmatrix} u_{d,i}(k) \\ u_{p,i}(k) \end{bmatrix}, \quad (14)$$

where the weights $R_{ii} \in \mathbb{R}$ and $Q_{ii} \in \mathbb{R}^{2 \times 2}$ indicate the relative importance of each agent. A hospital can be given more importance than a household, and one electric device might be more costly to operate than another electric device.

The network objective $V(x(k), u(k))$ at time k is assumed to be the sum of the individual objectives

$$V(x(k), u(k)) = \sum_{i=1}^n V_i(x_i(k), u_i(k)), \quad (15)$$

which is assumed to be time invariant. The centralized MPC problem is to minimize (15) over a prediction horizon, subject to the network dynamics and constraints from the devices. See Fig. 5.

4.1. Distributed MPC

Here we describe the distributed MPC method that will be implemented with the network model presented in Section 2. This method is based on a dual decomposition technique for MPC with sub-gradient iterations [14]. The algorithm finds the optimal change in flexible power and solves the local minimization problem at each agent i , as described in Section 4.1.1, iteratively with the sub-gradient steps described in Section 4.1.2 at each iteration k of the MPC scheme. Thus, a *distributed MPC* is obtained.

The original information structure is preserved, and as a bonus the Lagrangian multipliers $\lambda(k)$, that are introduced, can be interpreted as a price reference [19,12]. However, the multiplier can be seen as a coordination incentive more than a price in a currency. It is useful to work with price like concepts when dealing with allocation problems [20]. Each agent i bases their decision on maximising their objective function. The iterative adjustment of the price is similar to a market equilibrium process where the price is adjusted to match supply and demand. In our model, we view the price as a weighing parameter for the distance to the equilibrium price for power, as all agents will have a Lagrangian multiplier equal to zero when the demand and production is in balance.

4.1.1. Decoupled minimization problems

To decouple the state Eq. (5) a new set of constraints are introduced

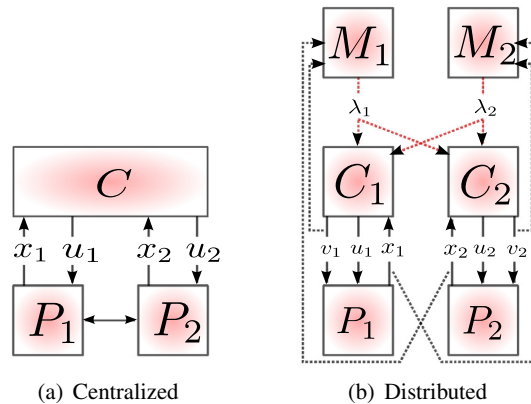


Fig. 5. A $n = 2$ agents schematic example, where the two agents share information with each other. In (a) the centralized case, the controller has to have full state information to coordinate all agents. In (b) the distributed case, each agent has its own local controller. However, a new coordination layer (the market makers) is present to coordinate the local controllers. The local controllers update the state and send it in the direction of the arrows in the information network A. The market makers update the prices and distribute the prices in the opposite direction of the arrows in the information network A. The solution of (a) and (b) is the same if the problem is convex.

$$v_i(\tau) = \sum_{j \in N_i} A_{ij} x_j(\tau), \quad (16)$$

which are interpreted as the influence agent i expects to receive from its neighboring agents $j \in N_i$. These constraints are relaxed by Lagrange relaxation, adding additional optimization variables $\lambda(k) = [\lambda_1(k), \dots, \lambda_n(k)]'$ to the problem.

In this section, the hat notation will represent prediction variables over the horizon T , and $\tau = k, \dots, k+T$ is a new time variable introduced to distinguish the prediction time steps from the system time steps k .

We solve a set of decoupled minimization problems (17) at each agent $i = 1, \dots, n$.

$$\begin{aligned} & \min_{\hat{u}_i} V_{\text{agent}}^i, \\ & \text{s.t. (19), (20) and (21) hold,} \end{aligned} \quad (17)$$

where we have defined

$$V_{\text{agent}}^i = \sum_{\tau=k}^{k+T} \left(V_i(\hat{x}_i(\tau), \hat{u}_i(\tau)) + \hat{\lambda}_i(\tau) \hat{v}_i(\tau) - \sum_{j \in N_i} \hat{\lambda}_j(\tau) A_{ji} \hat{x}_i(\tau) \right), \quad (18)$$

The problems (17) only depend on local measurements of initial states

$$\hat{x}_i(\tau)_{\tau=k} = x_i(k), \quad \hat{p}_i(\tau)_{\tau=k} = p_i(k), \quad \hat{w}_i(\tau)_{\tau=k} = w_i(k), \quad (19)$$

and local range constraints for all $\tau = k, \dots, k+T$

$$\begin{aligned} \hat{x}_i(\tau) &\in X_i, \quad \hat{u}_i(\tau) \in U_i, \\ \hat{w}_i(\tau) &\in W_i, \quad \hat{p}_i(\tau) \in P_i(\tau), \end{aligned} \quad (20)$$

where the constraint set $P_i(\tau)$ is determined by the technical constraints from the devices, and X_i, U_i, W_i are convex sets. There are local dynamic prediction models corresponding to (5) and (2) for all $\tau = k, \dots, k+T$

$$\begin{aligned} \hat{x}_i(\tau+1) &= A_{ii} \hat{x}_i(\tau) + \hat{v}_i(\tau) + B_{ii} \hat{u}_i(\tau) + \hat{w}_i(\tau), \\ \hat{d}_{f,i}(\tau+1) &= \hat{d}_{f,i}(\tau) + \hat{u}_{d,i}(\tau), \\ \hat{p}_{f,i}(\tau+1) &= \hat{p}_{f,i}(\tau) + \hat{u}_{p,i}(\tau). \end{aligned} \quad (21)$$

Different models for the change in external power $\hat{w}_i(k)$ can be included. This can be a forecast based on information from the agent or on historical data, or in the simplest case; the demand stays the same over the horizon. In addition (17) depends on price information $\hat{\lambda}_j$ from connected neighbors $j \in N_i$ in the information network.

4.1.2. Sub-gradient iterations

The Lagrange multipliers

$$\hat{\lambda}(\tau) = [\hat{\lambda}_1(\tau), \dots, \hat{\lambda}_n(\tau)]' \in \mathbb{R}^n \quad (22)$$

are introduced to relax the constraints (16). Their update coordinates the decoupled sub-problems presented in the previous Section 4.1.1.

For all $\tau = k, \dots, k+T$ the sub-gradient iterations of the prices are updated according to

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r} \left[\hat{v}_{i,r}(\tau) - \sum_{j \in N_i} A_{ij} \hat{x}_{j,r}(\tau) \right], \quad (23)$$

where r is the counter for the gradient iteration. In this way the price updates are also distributed, only depending on information from neighboring agents $j \in N_i$, and the gradient-steps $\gamma_{i,r}$ are chosen such that we converge to the optimum. The algorithm terminates when $\hat{v}_{i,r}(\tau) - \sum_{j \in N_i} A_{ij} \hat{x}_{j,r}(\tau)$ converges to zero, and hence (16) is met.

5. Simulation results

This paper focuses on the information sharing model of the network. We claim that we can use the network model presented in Section 2 both for coordinating supply side devices and demand side devices in the Smart Grid. In addition to the network model, we naturally need a model for the physical behavior of the devices. Therefore, as an illustration we here include a model of the devices present at the agents. We take the micro Combined Heat and Power (μ -CHP) systems to be the device to be controlled at all agents. This is a promising candidate for domestic power generation [21]. It can be fueled on gas, and it produces both heat and electric power to be used locally, when it is on. Due to the power output, the μ -CHP can both reduce demand to the grid and supply the grid with excess power.

A realistic model for the μ -CHP with heat-buffer is available in [22]. To illustrate how we include a device model with the network model, we here use a simple device model. We consider a maximum power output $p_{i,\max} > 0$ and a minimum output $p_{i,\min} = 0$, from the μ -CHP present at the end-users. Hence the production set is defined by,

$$P(k) = \{p_i(k) \mid -p_{i,\max} \leq p_i(k) \leq 0\}, \quad (24)$$

for all $k > 0$, which makes the overall problem to be solved convex. The production $p_i(k)$ at agent i is defined to be opposite in sign to the demand $d_i(k)$, see (5). We are interested in the imbalance in the network, and choose the demand to be positive and the production to be negative.

$$\begin{aligned} d_i(k) &\in \mathbb{R}_+ \\ p_i(k) &\in \mathbb{R}_-, \end{aligned} \quad (25)$$

Further, we model a one time-step delay between production and change in production. This represents the response time from the machine receives the control input, until we see it in the output of the system. The production $p_i(k)$ is related to the control input, change in production $u_i(k)$, in (5) by (2).

We show the results from simulations combining our proposed model presented in Section 2 for a network of $n = 150$ households with the method shown in Section 4. We allow the output from the μ -CHPs $p_i(k)$ to take any value between zero and $p_{\max} = 1 \text{ kW}$, and the predictions for the change in demand over the control horizon T is taken to be the measurement at $T = k$. Which means that $\hat{w}(\tau)_{\tau=k} = w_i(k)$ and $\hat{w}(\tau) = 0$ for $\tau = k + 1, \dots, k + T$. The simulations found by a GuRoBi QP-solver [23] version 4.6 with python 2.5. For details about the implementation see [24].

We use realistic power demand patterns [25] provided by the Energy research Centre of the Netherlands (ECN). The simulations use demand patterns from half a day in a November month, with a resolution of one minute. At this time of the year we can assume that the heat demand is high in the houses, in which case the heat production from the μ -CHPs is not wasted. Each of the 150 households have a unique demand pattern, which are variations of five different base user profiles.

We choose the information structure as in Fig. 4, only that the end users are also connected. Which means that we have a circular network consisting of 150 households. Such network for $n = 5$ is shown in Fig. 6. Each household weight their own information with 0.8 and two neighbors with 0.1. This means that if their own μ -CHP can provide the electric power needed in the household, most part of the production will be provided by their own μ -CHP. However, when the household has a power shortage, neighbors receive this information through the information network. The corresponding information matrix is given by

$$A = \begin{bmatrix} 0.8 & 0.1 & 0 & 0 & \dots & 0.1 \\ 0.1 & 0.8 & 0.1 & 0 & \dots & 0 \\ 0 & 0.1 & 0.8 & 0.1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & \dots & 0 & 0 & 0.1 & 0.8 \end{bmatrix}. \quad (26)$$

We perform simulations with the distributed scheme presented in Section 4.1, and use the following parameters: prediction horizon $T = 8$ minutes, gradient step size $\gamma_{i,r} = 0.4$, and maximum production $p_{\max} = 1 \text{ kW}$. These numbers are based on simulations. If the prediction horizon is too short we can not anticipate on future changes, and if it is too long the accuracy of the predictions decrease and the computation time increase. The gradient step is chosen small enough for the algorithm to converge, and the power output is a typical one for a domestic generator. The prediction for change in demand at a household i is based on the current measurement. It is predicted that the demand stays unchanged over the horizon, i.e.,

$$\hat{w}_{p,i}(k) = w_{p,i}(k) \quad (27)$$

$$\hat{w}_{p,i}(\tau) = 0, \quad \tau = k + 1, \dots, k + T. \quad (28)$$

The sub-gradient iterations described in Section 4.1 terminates when $|\hat{\lambda}_{i,r+1}(\tau) - \hat{\lambda}_{i,r+1}(\tau)| < 10$, for all $\tau = k, \dots, k + T$ and all $i = 1, \dots, 150$.

Fig. 7(a) shows the result for the network of 150 households. The blue stippled line shows the total demand in the network, and the red dotted line shows the production resulting from the distributed MPC algorithm. We see at $k = 400$, when the demand has a peak there is a corresponding peak in the production. The green solid line shows the imbalance in the net-

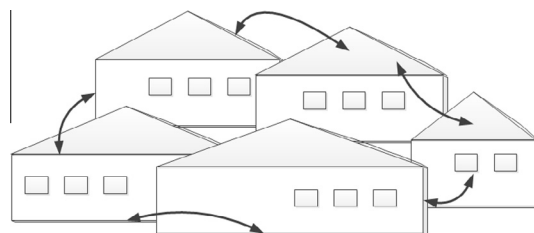


Fig. 6. Households with circular information structure as the A matrix (26), here shown for $n = 5$.

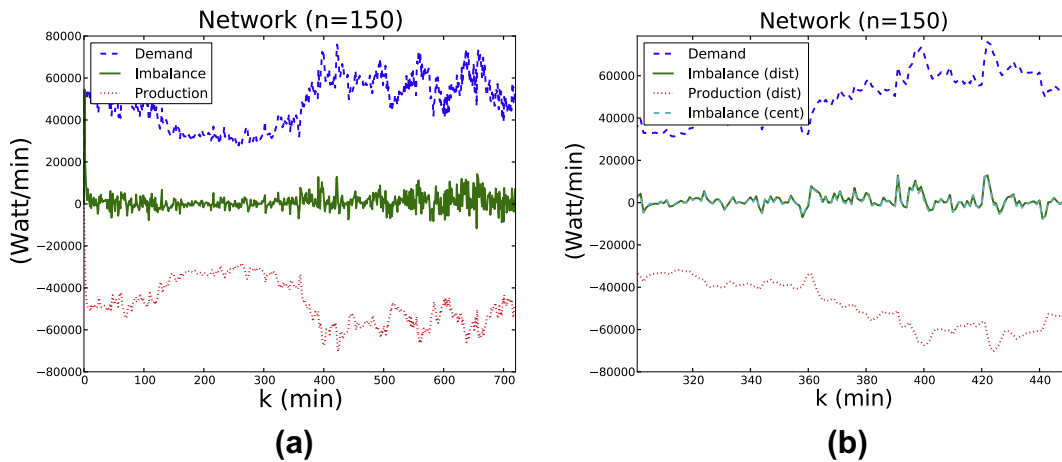


Fig. 7. Left: The net imbalance of the 150 households in the circular information network stays close to zero, which was target value in this simulation. When the imbalance is positive the network is a net consumer of power and when the imbalance is negative the network is a net producer of power. This shortage or excess of electric power is exchanged with the external network, and the objective function is formulated so that this exchange is minimized. Right: A zoomed in version of 7(a), which also including the centralized solution for comparison. We see that the imbalance in the distributed and the centralized case overlap.

work. We see that the imbalance $\sum_{i=1}^{150} x_i(k)$ stays close to zero, which means that the network can provide local production for local demand. This eases the stress on the external line. By including a more accurate forecast for the external demand, than the one implemented by (28), the fluctuations in the imbalance would be further minimized. Also notice that the network is a net producer when the imbalance is negative, and the network is a net consumer when the imbalance is positive.

Fig. 7(b) zooms in on the time range $k = 300, \dots, 450$. This figure includes the imbalance obtained with the centralized MPC controller in cyan stippled line. In this plot, we can not distinguish the central solution from the distributed solution. However, the distributed computation scales better than the centralized computation. We compare the computation time of a network of $n = 5$ households and $n = 150$ households. The centralized algorithm needs 29 times more time to solve for 150 households than 5 households, while the distributed algorithm uses 5 times longer per household to solve for 150 households than for 5 households. Results on degree of sub-optimality can be found in [14].

6. Discussion

In this paper, we have designed an information sharing model that can be used together with a distributed MPC method to achieve supply–demand balance in the power network. We couple the agents dynamically through their notion of power imbalance information. The model has the freedom to take into account different generator capacity on each agent, and the agents may be weighted with different importance. It is set up such that it can also be used together with different models for demand side and supply side devices, which is our currently on-going work. It is also possible to include storage, e.g. electric car batteries, to be coordinated using the network model described in Section 2. In Section 5, we show an example with control of μ -CHPs in a network. In [15,26] we have focused on the control challenges when including non-convex constraints from μ -CHP and washing machines in a network.

This research has raised many questions in need of further investigation. In Section 2 we have given some necessary conditions for the design of the information matrix A , and we have suggested possible network topologies. However, it would be interesting to assess the effects of different topologies. One side is the effect on the convergence speed of the algorithm, another side is to find an optimal information topology given a power grid topology. By choosing information neighbors wisely, transportation losses can be minimized. In addition, choosing the A matrix wisely, we can keep the load over the transformers close to a target value. This is beneficial for the network configuration, because it is cheaper when there can be more connections on each transformer. Therefore, it would be interesting to look for conditions for an optimal A matrix with respect to network losses, congestion over the transformer and convergence speed of the algorithm.

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